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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

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By B. Osgood Peirce.†

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Any one who has been compelled to measure the permeability of iron under high excitations, by experiments upon relatively short rods exposed to strong magnetizing fields, must have had occasion to notice that, under these circumstances, the corrections for the effects of the self-demagnetizing forces are often very small. In the case of an ellipsoidal specimen, it is possible to predict the phenomena which one encounters in practice,[1] and although it is not easy to make long ellipsoidal test rods with accuracy, there is a sufficiently close analogy between the two cases to make the general conclusions reached for ellipsoids applicable to the cylindrical rods usually employed in the laboratory.[2]

It is well known that if an ellipsoid of soft, magnetizable material and of fixed, constant susceptibility, k, were placed in a uniform magnetic field, H, it would become uniformly magnetized by induction and that the components of the field within it would be

$$H_x/(1+2\pi abckK_0), \quad H_y/(1+2\pi abckL_0), \quad H/(1+2\pi abckM_0),$$

where a, b, c, are the lengths of the semiaxes,

$$K_0 = \int_0^\infty \frac{ds}{(s+a)^{3/2}\cdot(s+b)^{1/2}\cdot(s+c)^{1/2}}, \tag{1}$$

and L_0, M_0, have corresponding values.

† Deceased January 14, 1914.

1 Rayleigh, Phil. Mag., **22**, 1886; Maxwell, Vol., II; Webster, Elect. and Mag., §§ 192, 196; Peirce, N. P. F.; Yosida, Tokyo, Proc., **3**, 8, 1906; Mallik, Phil. Mag., **14**, 1907; **15**, 1908; Daniele, N. Cimento, **2**, 6, 1911; H. E. J. G. duBois, Deutsch. Phys. Gesell., Verh. **15**, 8, 1913.

2 Ewing, Phil. Mag., **176**, 1885; Tanakadaté, Phil. Mag., **26**, 1888; H. E. J. G. duBois, Phil. Mag., **29**, 1890. Wied. Ann., **46**, 1892; Rossler, Elektrotechnische Zeitschrift, **14**, 1893; Mann, Dissertation Berlin, 1895. Phys. Rev., **3**, 1896; Ascoli e Lori, Rendic. R. Acad. d. Lincei, **3**: 2, 1894; **6**: 2, 1897; J. L. W. Gill, Phil. Mag., **46**, 1898; Morin, Écl. Électr. **15**, 1898; Holborn, Sitzungsbericht d. Akad. Wiss. zu Berlin, **1**, 1898; Benedicks, Wied. Ann., **6**, 1901; Bihang Svenska V.-Akad. Handlinger, **27**, 1902; H. E. J. G. duBois, Wied. Ann., **7**, 1902; Gumlich, Elektrotechnische Zeitschrift, **22**, 1901; **30**, 1909; Peirce, These Proceedings, **44**, 1908; Am. Journal of Science, **28**, 1909; Shuddemagen, These Proceedings, **43**, 1907; Peirce, Ibid., **49**, 1913.

If two of the semiaxes (b, c) were equal, and if the third axis had the direction of the outside field, H, which had been chosen also for the direction of the x axis, the field in the ellipsoid would agree with H in direction, and we might write,

$$K_0 = \frac{2}{e^2 \cdot a^3}\left(\frac{1}{2e} \cdot log\frac{1+e}{1-e} - 1\right), \tag{2}$$

where $e^2 = (a^2-b^2)/a^2 = (a^2-c^2)/a^2$. The ratio of the intensity of the field, H', within the ellipsoid to that of the exciting field, H, would be

$$\frac{1}{(1+2\pi abckK_0)} \tag{3}$$

Table I shows the numerical values of a^3K_0 and of $1 + 2\pi abckK_0$ for several different values of the ratio $m = a/b$.

TABLE I.

m	$a^3 \cdot K_0$	$1+2\pi abckK_0$
30	6.198	$1 + 0.0433k$
40	6.771	$1 + 0.0266k$
50	7.215	$1 + 0.0181k$
60	7.575	$1 + 0.0132k$
80	8.150	$1 + 0.0080k$
100	8.597	$1 + 0.0054k$
140	9.270	$1 + 0.0030k$
200	9.983	$1 + 0.0016k$

Even though it be not possible to realize these conditions exactly in practice, and a relatively slight departure from a truly ellipsoidal form in the testpiece may alter the conclusions appreciably, yet the measurements of several observers who have used such nearly ellipsoidal rods as they were able to procure, show a fairly close agreement with this theory, and we shall find it instructive to examine the numerical results obtained by applying it in the cases of one or two kinds of soft iron and steel to be bought in the market.

Some time ago, Mr. J. Coulson and I examined with great care a long rod of soft Bessemer steel in a uniformly wound solenoid of 20904 turns, 4.85 meters long, as well as shorter specimens of the same material between the poles of a massive soft iron yoke. Table II gives very approximately for this steel and for various values of H, the values of the susceptibility; of the fractional increase in the

induction, B, due to an increase of one unit in the exciting field, H, in the metal; and of the fractional change in B due to a change in H of one percent of its own value. These quantities are denoted by k, X, Z, respectively.

TABLE II.

H	k	X	Z
30	38.3	0.00558	0.0017
40	30.1	0.00386	0.0016
50	24.9	0.00277	0.0014
60	21.3	0.00227	0.0014
80	16.5	0.00155	0.0013
100	13.6	0.00128	0.0013
120	11.0	0.00108	0.0013
160	9.0	0.00082	0.0013
200	7.4	0.00067	0.0013
300	5.2	0.00046	0.0013
400	4.1	0.00032	0.0013
500	3.3	0.00024	0.0012
800	2.2	0.00014	0.0011
1000	1.7	0.00011	0.0011
1500	1.1	0.00005	0.0008
2000	0.9	0.00004	0.0009
2500	0.7	0.00004	0.0010
(5000)	(0.34)	(0.00004)	(0.0019)

From the numbers in the last column of Table I, and those in the second column of Table II, it is easy to compute the ratio of the intensities of the magnetizing field (H') in the metal, and the external exciting field (H), for different values of m. The results of this procedure for $a/b = 30$, a/b, $= 50$, and $a/b = 100$, appear in Table III.

TABLE III.

H'	H'/H, if $a/b = 30$,	H'/H, if $a/b = 50$,	H'/H, if $a/b = 100$
30	0.377	0.591	0.828
40	0.435	0.648	0.861
50	0.482	0.689	0.882
60	0.521	0.718	0.897
80	0.584	0.770	0.918
100	0.630	0.803	0.932

TABLE III — *Continued.*

H'	H'/H, if $a/b = 30$,	H'/H, if $a/b = 50$,	H'/H, if $a/b = 100$
120	0.666	0.826	0.941
160	0.720	0.862	0.955
200	0.758	0.880	0.962
300	0.816	0.914	0.973
400	0.850	0.933	0.978
500	0.876	0.943	0.982
800	0.913	0.962	0.988
1000	0.932	0.970	0.991
1500	0.955	0.981	0.994
2000	0.963	0.984	0.995
2500	0.970	0.987	0.996
(5000)	(0.985)	(0.994)	(0.998)

A comparison between the figures given in this table and the values of Z in Table II shows that, in the case of an ellipsoidal rod only 30 diameters long, the density of the flux of magnetic induction through the metal when the exciting field is as high as 2500 gausses, is not so much as one third of one per cent less than the corresponding flux density for an infinitely long rod. When the ellipsoid is 50 diameters long, the flux density for 2500 gausses does not differ by so much as one eighth of one per cent from the flux density in an infinitely long specimen under the same excitation, and only an extremely good determination of B is correct within this fraction when the value of B is above 25000.

It is interesting to note that if, under excitations above, say, 2500 gausses, we may assume the intensity of magnetization, $I = kH$, to be constant, so that $B - H$ is constant; dB/dH is unity and $\left(\frac{H}{B} \cdot \frac{dB}{dH}\right)$ increases somewhat with H.

In the case of the Bessemer steel mentioned here, where $I_\infty = 1694$, Z is 0.00190, for $H = 5000$, while Z is 0.00484, if H is 20000.

Table IV gives results obtained by Mr. Coulson and myself from a long series of tests upon a special brand of Norway Iron unusually permeable at high excitations.

TABLE IV.

H.	k.	Z.	$(H'/H)_{30}$	$(H'/H)_{50}$
400	4.4	0.0010	0.841	0.922
500	3.5	0.0009	0.869	0.940
600	2.9	0.0006	0.889	0.950
700	2.4	0.0005	0.909	0.957
800	2.2	0.0004	0.913	0.961
900	1.9	0.0004	0.926	0.965
1000	1.8	0.0005	0.931	0.968
1500	1.2	0.0006	0.951	0.978
2000	0.9	0.0008	0.962	0.984
2500	0.7	0.0010	0.970	0.987

In the case of this iron, the flux density, under an excitation of 2500 gausses, would be less than for an infinitely long rod, by about one third of one per cent, for $a/b = 30$, and, by about one eighth of one per cent, for $a/b = 50$.

It has long been known that, although cylindrical rods of soft iron do not become uniformly magnetized when they are exposed longitudinally to uniform magnetizing fields, yet they behave in many other respects much like the ellipsoidal pieces which are more easily subjected to analysis.

A glance at the B vs. H curves obtained thirty years ago by Ewing from an iron wire 0.158 cms. in diameter and originally 47.5 cms. long, shows that under very high excitations the flux through the central cross section of a comparatively short piece of this wire would have been much the same as the corresponding flux through a specimen several hundred diameters long, but the demagnetizing factor in the case of such a rod seems to be a function of both the diameter and the length and not a function of the ratio of the two alone, and although at low excitations the numbers given by DuBois and by Shuddemagen are most useful, it is not easy to compute with certainty just how long a specimen must be in order that the flux through its meridian section may be assumed to be unaffected by the nearness of the ends of the rod. It happens that Mr. John Coulson and I, who have been studying the maximum value of I in different kinds of iron, have had occasion to determine the magnitude of the influence of the ends of some rods which we have been using in fields of 2500 gausses and upwards, and we have found that we might have used much shorter test pieces and a much less massive solenoid than we have

employed in our work. Some of our observations are recorded briefly here in the hope that they may prove useful to other experimenters.

Figure 1 shows the general arrangement of the apparatus used in making the measurements described in this paper. Figure 2 indicates the forms of three of the several standards of mutual inductances which we used for calibrating the galvanometers. For a complete discussion of the apparatus used throughout this work reference is made to a preceding paper,[3] since the apparatus used for both investigations was essentially the same.

Our method of work was not new in any particular, but the necessity of using heavy currents and, therefore, of taking care of the heat equivalent of many kilowatts in our circuit, and the use of ballistic galvanometers with periods so long that a reversal of current in the highly inductive circuit could be accomplished before the galvanometer coil had moved appreciably from its position of rest, introduced many difficulties which could be overcome only after much anxious experimentation.

The large solenoid with which most of our work at high excitations was done, was made of about 300 kilograms of triply covered No. 10 copper wire wound uniformly, with great care, by Mr. George W. Thompson, the mechanician of the Jefferson Laboratory, upon a massive brass spool, 186.2 centimeters long, in inside measurements. There are two coils, one of 8117 turns in 14 layers, and the other, of slightly different wire, of 5872 turns in 10 layers. The field intensity in the centre of the solenoid when a current of one ampere passes through the first coil is 54.71 gausses, while at a point fifty centimeters from the centre on the axis, the intensity is 54.60 gausses. A current of one ampere sent through both coils in series creates a field of 94.19 gausses at the centre of the coil. This solenoid was repeatedly tested for leakage between the turns by means of a very carefully made test coil without iron, but we were never able to detect any evidence of fault.

For low excitations, we sometimes used a somewhat longer solenoid wound with No. 14 wire.

The test pieces were first packed in fine iron filings in a pipe closed at the ends by caps. This was placed horizontal and perpendicular to the meridian upon supports in a furnace where it was exposed to several hundred gas jets driven by a power compressor. The dimen-

[3] Peirce, These Proceedings, **49**, 1913.

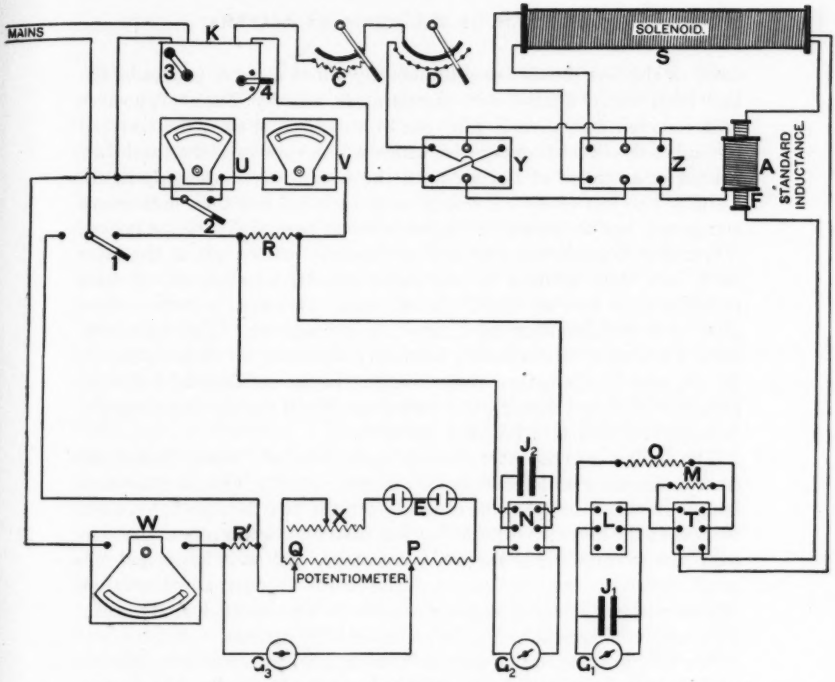

FIGURE 1. Shows diagrammatically the general arrangement of some of the apparatus used in making the observations described in this paper.

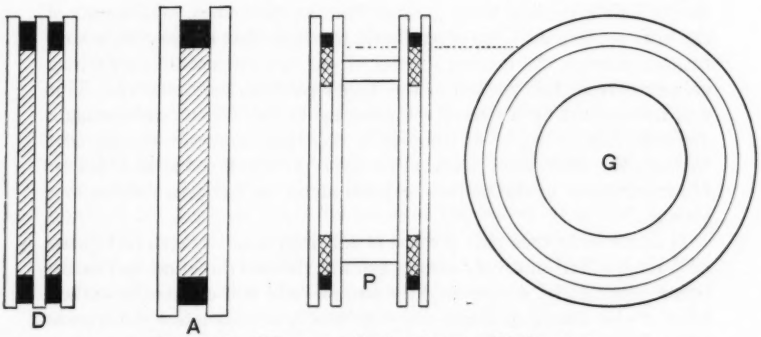

FIGURE 2. Three standards of mutual inductance.

sions of the test pieces were obtained by aid of a set of gauges which had been tested against two comparators, one by Zeiss. The test rods received first a very thin coat of varnish and then the test coil of triply silk-covered copper wire which was in turn varnished and then heated in a stream of hot air until the whole was thoroughly dried. Usually two test coils were wound upon each rod and their indications compared, lest an injury or imperfection in one might escape notice.

Our first experiments were not so carefully carried out as the later ones, but they pointed to the same general conclusions. I need mention only two of them. In the first, we tested a rather short piece of cold-rolled shafting 1.269 cms. in diameter. This had originally a length of 79 diameters, but was cut shorter by steps to 63, 47, 32, 24, and 16 diameters, respectively. In an exciting field of 1280 gausses, the fluxes through the central section of the specimen seemed to be as 176, 175, 175, 175, 175, and 175.

The second experiment was made upon a rod of Norway Iron 1.110 cms. in diameter and successively 150, 130, 110, 70, and 30 diameters long. In this case the value obtained for the flux through the central section at the last step differed by less than one eighth of one per cent from the average value for the other steps, and as it happened this small difference was in excess. The truth is that all the values agreed within the small accidental error to be expected in the work. The magnetizing field had an intensity of 2700 gausses.

We were at first puzzled by a phenomenon which sometimes affected our results by a small fraction of one per cent. After a long day's work when the originally long rod under examination had been cut down, by a succession of steps, to a short one, and the resistance of the circuit had become a little greater than at the outset on account of the heat set free in it, we often found the same flux at very high excitations through the central section of our specimen which a slightly greater current had caused in the longer piece in the morning. This was not caused by a rise of temperature in the test piece because a vigorous flow of water of practically constant temperature was sent through the tube of the solenoid all day. It was not due to a change of sensitiveness in the ballistic galvanometer, as frequent calibrations showed.

It seems to be true that if a rod of soft iron of any length be repeatedly magnetized in a very strong field, in the one direction and in the other, alternately, a very slightly weaker field will eventually suffice, after it also has been many times reversed, to magnetize the iron as strongly as the original one.

We thought it possible that we might avoid this difficulty by using at the same time a long piece and a short piece, cut freshly from a single rod, but experiment seemed to show that we could not expect the same permeability in two pieces obtained in this way and that the difference might easily be as much as one per cent at some excitations. We believe, however, that we have reduced the error in our conclusions due to the effect just mentioned to a very small fraction.

All our work, which lasted more than two months, pointed to the same results, and I need mention only a few representative experiments. They convinced us that in our determinations of the saturation values of the magnetization in different kinds of iron and steel, we might safely use much shorter test pieces than those which we had employed before.

Experiment *A* was made upon an annealed piece of Bessemer rod 0.635 cms. in diameter. The original length was 158 diameters, but this was cut down successively to 61, 31, and 18 diameters. At an excitation of 2600 gausses where *B* was 23530, the average of the fluxes of magnetic induction through the central sections of the various pieces, differed by less than one sixth of one per cent from any one of the individual values, and from the last value by a wholly inappreciable fraction.

Experiment *B* was made upon a Bessemer rod 0.795 cms. in diameter. The lengths of the pieces were 150, 100, 60, 40, and 20 cms. For *H* 1320, the flux through the central section of the shortest piece seemed to be about one fifth of one per cent lower than the average of the corresponding fluxes for the other pieces. At 1850 gausses and 2700 gausses, however, there was no such difference. For *H* 2570, *B* was 23700. Figure 3 shows some *B*–*H* curves for the pieces at low excitations where the effects of the presence of the ends of the rods are very apparent.

In Experiment *C*, we treated a somewhat stouter Bessemer rod. Its diameter was 0.879 cms. Its length was originally 164 diameters, but this was shortened by steps to 124, 100, 75, 45, and 24 diameters. At 2500 gausses the value of the flux through the centre of a piece only 19 cms., or 24 diameters, long, did not differ by so much as one tenth of one per cent from the mean of the fluxes for all the lengths under this excitation, where *B* was 23550. At an excitation of 1710 gausses, *B* was 22700, and it was not possible to prove that the shortest piece had a less flux than the others. Some of the results of our observations for this experiment are tabulated in Table V. *H* is the value that the field would have inside the solenoid if the iron were

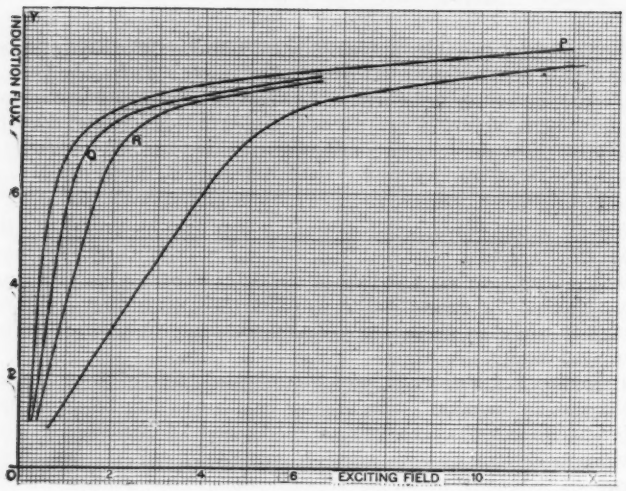

FIGURE 3 represents the B–H curves for four lengths of the same material, (experiment B). The curves OP, OQ, OR, and OS correspond to specimens of lengths 150, 60, 40, and 20 cms. respectively, plotted on an arbitrary scale. One unit on the Y axes corresponds to a B of 6000 gausses; and one unit on the X axes to a field H of 12.17 gausses.

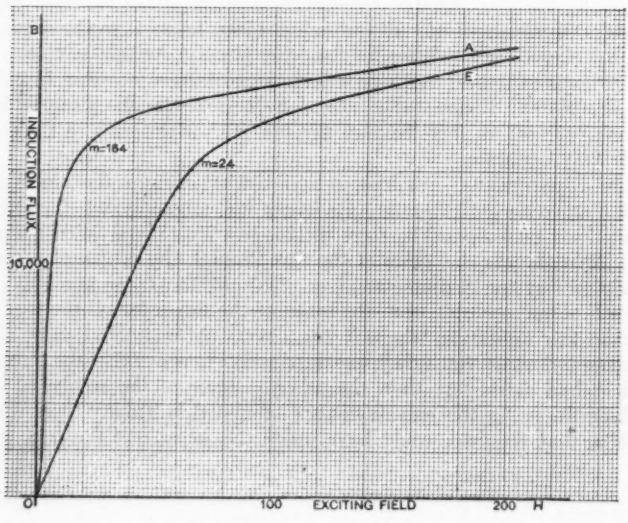

FIGURE 4 shows the B–H curves for two of the specimens used in experiment C. These curves OA and OE are plotted from Table V, for $m = 164$ and $m = 24$.

TABLE V.

$m = 164$		$m = 124$		$m = 75$		$m = 45$		$m = 24$	
H	B	H	B	H	B	H	B	H	B
1.00	790								
2.70	2980								
3.92	6020								
5.35	8930	5.27	7250	5.27	4600	5.23	1593	5.23	1000
5.90	9770	7.67	10640	7.67	7250	7.61	4240	7.63	1600
7.72	11330	8.60	11600	8.40	8400	9.80	5600	9.60	2000
10.50	13330	9.60	12200	9.80	10000	11.8	7000	11.56	2480
17.00	13760	11.56	13320	11.56	11030	13.6	8400	13.5	3000
14.80	14440	14.00	14090	13.50	12200	14.9	9410	15.0	3400
18.90	15050	16.92	14620	16.85	13820	17.2	10800	16.8	3780
22.30	15380	20.00	15000	21.97	14980	21.8	12590	21.9	5000
24.75	15550	24.10	15420	26.0	15400	30.0	14840	32.8	7800
43.7	16430	42.6	16430	43.1	16490	42.6	16050	42.6	10120
85.1	17400	78.0	16920	52.0	16720	52.0	16400	60.0	13460
110.5	17890	106.0	17800	70.0	17140	70.0	16960	85.0	15700
127.0	18200	122.0	18100	113.9	17960	100.0	17600	100.0	16400
138.0	18400	136.0	18360	136.0	18380	136.0	18280	118.0	17000
149.0	18600							140.0	17600
173.0	19000							156.0	18000
209.5	19240							174.0	18360
298.0	20200							190.0	18600
441.0	20890							203.8	18810
603.0	21450							413.	20710
824.0	21660							571.	21350
1066.	21970							1014.	21950
1235.	22240							1190.	22230
1794.	22770							1700.	22680
2600.	23590	2540	23570	2440	23600	2481.	23590	2492.	23550

absent, B the induction flux through the central cross section of the rod under investigation, and m denotes the ratio of the length of the rod to its diameter. Figure 4 gives two B–H curves for this material.

In Experiment D, we studied the behavior of a rod of fine annealed tool steel 0.794 cms. in diameter for which the final value of the magnetization vector, I, is about 1540. This was furnished with a test coil of 21 turns wound about its centre and was tested first when it had a length of 91.3 cms. and again when it had been shortened to 21.8 cms. At excitations of 1300, 1800, and 2650 gausses the fluxes did not differ from each other by more than one fifth of one per cent of either. At 900 gausses, however, the shorter rod had a flux quite one per cent below that of the longer one. At 2700 gausses, the value of B was about 21960.

A rod of Special Magnet Steel, 0.805 cms. in diameter and originally 124 diameters long, gave upon being shortened to about 30 diameters, one half of one per cent less flux in a field of 1260 gausses, but in fields of about 1750 and 2600 gausses the fluxes in the case of the short piece were indistinguishable from the corresponding fluxes for the longer one. This was Experiment E.

It seems to be true, therefore, that in properly constructed solenoids with uniform fields above 2500 gausses, we may safely use rods only 25 or 30 diameters long with the expectation of finding that the fluxes through their meridian sections are the same, within the limits of ordinary careful laboratory practice, as if the pieces were infinitely long. At low excitations, however, the corrections for the ends of pieces as short as this would, of course, be very large. The magnetizing solenoid should be about 25 diameters of its own coil longer than the test piece. For ordinary work, where an accuracy of one half of one per cent will suffice, test pieces only 15 diameters long will often serve at excitations above 2500 gausses.

I wish to express my great obligation to the Trustees of the Bache Fund of the National Academy of Sciences for the loan of some of the apparatus used in making the observations mentioned in this paper.

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